## Exam Graph Theory

## 23 January 2023, 15.00-17.00

- It is absolutely not allowed to use calculators, phones, computers, books, notes, the help of others or any other aids.
- Always make sure to state clearly any results from the lecture notes you are using.
- Write the answer to each question on a separate sheet, with your name and student number on each sheet. This is worth 10 points (out of a total of 100).


## Exercise 1 ( 20 pts).

Determine the result of the Gale-Shapley algorithm on the following situation:

| boys | girls |
| :---: | :---: |
| $b_{1}: g_{1}>g_{2}>g_{4}>g_{3}$ | $g_{1}: b_{4}>b_{1}>b_{2}>b_{3}$ |
| $b_{2}: g_{2}>g_{1}>g_{3}>g_{4}$ | $g_{2}: b_{3}>b_{4}>b_{2}>b_{1}$ |
| $b_{3}: g_{3}>g_{2}>g_{4}>g_{1}$ | $g_{3}: b_{2}>b_{4}>b_{3}>b_{1}$ |
| $b_{4}: g_{3}>g_{1}>g_{4}>g_{2}$ | $g_{4}: b_{3}>b_{1}>b_{4}>b_{2}$ |

Make sure to clearly indicate, for each step of the algorithm, what actions are taken by the algorithm.

## Exercise 2 (20 pts)

Apply Kruskal's algorithm to find a minimum spanning tree in the following edge-weighted graph.


Again, make sure to clearly indicate, for each step of the algorithm, what actions are taken by the algorithm.

## Exercise 3 (20 pts)

Determine a maximum matching in the following graph, and give a short proof that your answer is correct.


Exercise 4 ( 30 pts).
Let $G$ be a cycle.
a) Show that

$$
\chi(G)= \begin{cases}2 & \text { if } v(G) \text { is even } \\ 3 & \text { if } v(G) \text { is odd }\end{cases}
$$

b) Show that $\chi_{\ell}(G)=\chi(G)$.

