EXAM GRAPH THEORY

23 January 2023, 15.00-17.00

- It is absolutely not allowed to use calculators, phones, computers, books, notes, the help of others or any other aids.
- Always make sure to state clearly any results from the lecture notes you are using.
- Write the answer to each question on a separate sheet, with your name and student number on each sheet. This is worth 10 points (out of a total of 100).

Exercise 1 (20 pts).

Determine the result of the Gale-Shapley algorithm on the following situation:

boys	girls
$b_1: g_1 > g_2 > g_4 > g_3$	$g_1: b_4 > b_1 > b_2 > b_3$
$b_2: g_2 > g_1 > g_3 > g_4$	$g_2: b_3 > b_4 > b_2 > b_1$
$b_3: g_3 > g_2 > g_4 > g_1$	$g_3: b_2 > b_4 > b_3 > b_1$
$b_4: g_3 > g_1 > g_4 > g_2$	$g_4: b_3 > b_1 > b_4 > b_2$

Make sure to clearly indicate, for each step of the algorithm, what actions are taken by the algorithm.

Exercise 2 (20 pts)

Apply Kruskal's algorithm to find a minimum spanning tree in the following edge-weighted graph.



Again, make sure to clearly indicate, for each step of the algorithm, what actions are taken by the algorithm.

Exercise 3 (20 pts)

Determine a maximum matching in the following graph, and give a short proof that your answer is correct.



Exercise 4 (30 pts). Let G be a cycle.

a) Show that

$$\chi(G) = \begin{cases} 2 & \text{if } v(G) \text{ is even,} \\ 3 & \text{if } v(G) \text{ is odd.} \end{cases}$$

b) Show that $\chi_{\ell}(G) = \chi(G)$.